



# INDIANA COMMON CORE STATE STANDARDS GEOMETRY

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## Introduction

The world we live in continues to change. For students to succeed in school, at work, and in the community, they will need more skills and knowledge than ever before. To ensure all students have every opportunity to succeed, Indiana adopted the Common Core State Standards in the area of Mathematics, as well as English Language Arts and Literacy in History/Social Studies, Science, and Technical Subjects.

## Common Core State Standards Philosophy

The Common Core State Standards:

- are aligned with college and work expectations.
- include rigorous content *and* application of knowledge through high-order skills.
- build upon strengths and lessons of current state standards.
- are internationally benchmarked.
- are evidence and/or research-based.

The Common Core State Standards will

- help prepare all students with the knowledge and skills needed to succeed in college and careers.
- assist students who move between states.
- create the opportunity for America to compete for high-wage, high-skill jobs in a knowledge-based economy.
- allow for more focused pre-service education and professional development.
- create economies of scale around areas such as curriculum, instructional resources, and assessment.

## Role of the Teacher

While the standards set expectations for student learning for each grade level, they do not prescribe how the standards should be taught. Teachers should use their skills, experience, talents, and resources to design standards-based classroom lessons that meet the needs of individual students.

## Assessment Horizon

Indiana is a governing state in the Partnership for Assessment of Readiness for College and Careers (PARCC). Through PARCC a new generation of assessments will be developed and used throughout the nation, allowing us to compare and evaluate policies that affect student achievement across states and school districts.

The Common Core assessment includes a summative test, which will be administered online beginning with the 2014-15 school year. The assessment also includes “through-course assessments” that take place over the course of the year, allowing teachers to adjust their instruction based on students’ results. As a governing state, Indiana will pilot Common Core items in 2012-13 and 2013-14 and provide schools with data regarding student performance on the Common Core State Standards.

## Impact on Instruction

### Grades K, 1, and 2

In 2011-12, kindergarten teachers will be the first to teach *only* the Common Core. Those kindergarteners will be the first class of 3<sup>rd</sup> graders to participate in the Common Core assessment. First-grade and second-grade students will learn Indiana Academic Standards and identified elements of Common Core in 2011-12. First-grade students will learn *only* Common Core in 2012-13. Second-grade students will learn *only* Common Core in 2013-14.

### Grades 3-12

Students in grades 3-12 will receive an intentional introduction to the Common Core beginning with the 2011-12 school year to ensure they have the foundation for future learning with no instructional gaps. In 2014-15, all ELA and math teachers will *only* teach Common Core.

## Final Notes

Indiana Department of Education staff members are working diligently, both externally with other states, and internally across the IDOE, to ensure the development of a common scope and sequence for Common Core. This work is critical as we begin the transition to the Common Core State Standards.

Equally as important, efforts are underway to assist teachers in understanding what to teach next year (and in subsequent years). A tight alignment between the content which is “taught” and that which is “tested” on ISTEP+ and ECAs, together with the initiation of Common Core State Standards instruction, facilitates the important work of developing the foundation for future learning and assessment.

## Traditional Pathway: Geometry

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas, organized into six units are as follows.

**Critical Area 1:** In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

**Critical Area 2:** Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

**Critical Area 3:** Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

**Critical Area 4:** Building on their work with the Pythagorean theorem in 8<sup>th</sup> grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

**Critical Area 5:** In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

**Critical Area 6:** Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Units	Includes Standard Clusters*	Mathematical Practice Standards
<b>Unit 1</b> Congruence, Proof, and Constructions	<ul style="list-style-type: none"> <li>Experiment with transformations in the plane.</li> <li>Understand congruence in terms of rigid motions.</li> <li>Prove geometric theorems.</li> <li>Make geometric constructions.</li> </ul>	<b>Make sense of problems and persevere in solving them.</b>  <b>Reason abstractly and quantitatively.</b>  <b>Construct viable arguments and critique the reasoning of others.</b>  <b>Model with mathematics.</b>  <b>Use appropriate tools strategically.</b>  <b>Attend to precision.</b>  <b>Look for and make use of structure.</b>  <b>Look for and express regularity in repeated reasoning.</b>
<b>Unit 2</b> Similarity, Proof, and Trigonometry	<ul style="list-style-type: none"> <li>Understand similarity in terms of similarity transformations.</li> <li>Prove theorems involving similarity.</li> <li>Define trigonometric ratios and solve problems involving right triangles.</li> <li>Apply geometric concepts in modeling situations.</li> <li>Apply trigonometry to general triangles.</li> </ul>	
<b>Unit 3</b> Extending to Three Dimensions	<ul style="list-style-type: none"> <li>Explain volume formulas and use them to solve problems.</li> <li>Visualize the relation between two-dimensional and three-dimensional objects.</li> <li>Apply geometric concepts in modeling situations.</li> </ul>	
<b>Unit 4</b> Connecting Algebra and Geometry through Coordinates	<ul style="list-style-type: none"> <li>Use coordinates to prove simple geometric theorems algebraically.</li> <li>Translate between the geometric description and the equation for a conic section.</li> </ul>	
<b>Unit 5</b> Circles With and Without Coordinates	<ul style="list-style-type: none"> <li>Understand and apply theorems about circles.</li> <li>Find arc lengths and areas of sectors of circles.</li> <li>Translate between the geometric description and the equation for a conic section.</li> <li>Use coordinates to prove simple geometric theorems algebraically.</li> <li>Apply geometric concepts in modeling situations.</li> </ul>	
<b>Unit 6</b> Applications of Probability	<ul style="list-style-type: none"> <li>Understand independence and conditional probability and use them to interpret data.</li> <li>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</li> <li>Use probability to evaluate outcomes of decisions.</li> </ul>	

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

## Unit 1: Congruence, Proof, and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Unit 1: Congruence, Proof, and Constructions	
Clusters and Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Experiment with transformations in the plane.</li> </ul> <p><i>Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.</i></p>	<p>G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</p> <p>G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).</p> <p>G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.</p> <p>G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.</p> <p>G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.</p>
<ul style="list-style-type: none"> <li>Understand congruence in terms of rigid motions.</li> </ul> <p><i>Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.</i></p>	<p>G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.</p> <p>G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.</p> <p>G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.</p>
<ul style="list-style-type: none"> <li>Prove geometric theorems.</li> </ul> <p><i>Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 in Unit 5.</i></p>	<p>G.CO.9 Prove theorems about lines and angles. <i>Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</i></p> <p>G.CO.10 Prove theorems about triangles. <i>Theorems include: measures of interior angles of a triangle sum to <math>180^\circ</math>; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</i></p> <p>G.CO.11 Prove theorems about parallelograms. <i>Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</i></p>

Unit 1: Congruence, Proof, and Constructions	
Clusters and Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Make geometric constructions.</li> </ul> <p><i>Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects.</i></p> <p><i>Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.</i></p>	<p>G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). <i>Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</i></p> <p>G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.</p>

## Unit 2: Similarity, Proof, and Trigonometry

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

Unit 2: Similarity, Proof, and Trigonometry	
Clusters and Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Understand similarity in terms of similarity transformations.</li> </ul>	<p>G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor.</p> <ul style="list-style-type: none"> <li>a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.</li> <li>b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</li> </ul> <p>G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</p> <p>G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</p>
<ul style="list-style-type: none"> <li>Prove theorems involving similarity.</li> </ul>	<p>G.SRT.4 Prove theorems about triangles. <i>Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</i></p> <p>G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</p>
<ul style="list-style-type: none"> <li>Define trigonometric ratios and solve problems involving right triangles.</li> </ul>	<p>G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</p> <p>G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.</p> <p>G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*</p>
<ul style="list-style-type: none"> <li>Apply geometric concepts in modeling situations.</li> </ul> <p><i>Focus on situations well modeled by trigonometric ratios for acute angles.</i></p>	<p>G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*</p> <p>G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*</p> <p>G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*</p>
<ul style="list-style-type: none"> <li>Apply trigonometry to general triangles.</li> </ul> <p><i>With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.</i></p>	<p>G.SRT.9 (+) Derive the formula <math>A = \frac{1}{2} ab \sin(C)</math> for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</p> <p>G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.</p> <p>G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</p>



### Unit 3: Extending to Three Dimensions

Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Unit 3: Extending to Three Dimensions	
Clusters and Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Explain volume formulas and use them to solve problems.</li> </ul> <p><i>Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor <math>k</math>, its area is <math>k^2</math> times the area of the first. Similarly, volumes of solid figures scale by <math>k^3</math> under a similarity transformation with scale factor <math>k</math>.</i></p>	<p>G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. <i>Use dissection arguments, Cavalieri's principle, and informal limit arguments.</i></p> <p>G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*</p>
<ul style="list-style-type: none"> <li>Visualize the relation between two-dimensional and three-dimensional objects.</li> </ul>	<p>G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</p>
<ul style="list-style-type: none"> <li>Apply geometric concepts in modeling situations.</li> </ul> <p><i>Focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles.</i></p>	<p>G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*</p>

## Unit 4: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean theorem in 8<sup>th</sup> grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Unit 4: Connecting Algebra and Geometry Through Coordinates	
Clusters and Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Use coordinates to prove simple geometric theorems algebraically.</li> </ul> <p><i>This unit has a close connection with the next unit. For example, a curriculum might merge G.GPE.1 and the Unit 5 treatment of G.GPE.4 with the standards in this unit. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.</i></p> <p><i>Relate work on parallel lines in G.GPE.5 to work on A.REI.5 in High School Algebra I involving systems of equations having no solution or infinitely many solutions.</i></p> <p><i>G.GPE.7 provides practice with the distance formula and its connection with the Pythagorean theorem.</i></p>	<p>G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. <i>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point <math>(1, \sqrt{3})</math> lies on the circle centered at the origin and containing the point <math>(0, 2)</math>.</i></p> <p>G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).</p> <p>G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.</p> <p>G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★</p>
<ul style="list-style-type: none"> <li>Translate between the geometric description and the equation for a conic section.</li> </ul> <p><i>The directrix should be parallel to a coordinate axis.</i></p>	<p>G.GPE.2 Derive the equation of a parabola given a focus and directrix.</p>

## Unit 5: Circles With and Without Coordinates

In this unit, students prove basic theorems about circles, with particular attention to perpendicularity and inscribed angles, in order to see symmetry in circles and as an application of triangle congruence criteria. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or parabolas and between two circles.

Unit 5: Circles With and Without Coordinates	
Clusters and Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Understand and apply theorems about circles.</li> </ul>	<p>G.C.1 Prove that all circles are similar.</p> <p>G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. <i>Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.</i></p> <p>G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.</p> <p>G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.</p>
<ul style="list-style-type: none"> <li>Find arc lengths and areas of sectors of circles.</li> </ul> <p><i>Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.</i></p>	<p>G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</p>
<ul style="list-style-type: none"> <li>Translate between the geometric description and the equation for a conic section.</li> </ul>	<p>G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</p>
<ul style="list-style-type: none"> <li>Use coordinates to prove simple geometric theorems algebraically.</li> </ul> <p><i>Include simple proofs involving circles.</i></p>	<p>G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. <i>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point <math>(1, \sqrt{3})</math> lies on the circle centered at the origin and containing the point <math>(0, 2)</math>.</i></p>
<ul style="list-style-type: none"> <li>Apply geometric concepts in modeling situations.</li> </ul> <p><i>Focus on situations in which the analysis of circles is required.</i></p>	<p>G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*</p>

## Unit 6: Applications of Probability

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Unit 6: Applications of Probability	
Clusters and Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Understand independence and conditional probability and use them to interpret data.</li> </ul> <p><i>Build on work with two-way tables from Algebra I Unit 3 (S.ID.5) to develop understanding of conditional probability and independence.</i></p>	<p>S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</p> <p>S.CP.2 Understand that two events <math>A</math> and <math>B</math> are independent if the probability of <math>A</math> and <math>B</math> occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</p> <p>S.CP.3 Understand the conditional probability of <math>A</math> given <math>B</math> as <math>P(A \text{ and } B)/P(B)</math>, and interpret independence of <math>A</math> and <math>B</math> as saying that the conditional probability of <math>A</math> given <math>B</math> is the same as the probability of <math>A</math>, and the conditional probability of <math>B</math> given <math>A</math> is the same as the probability of <math>B</math>.</p> <p>S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i></p> <p>S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i></p>
<ul style="list-style-type: none"> <li>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</li> </ul>	<p>S.CP.6 Find the conditional probability of <math>A</math> given <math>B</math> as the fraction of <math>B</math>'s outcomes that also belong to <math>A</math>, and interpret the answer in terms of the model.</p> <p>S.CP.7 Apply the Addition Rule, <math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math>, and interpret the answer in terms of the model.</p> <p>S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, <math>P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)</math>, and interpret the answer in terms of the model.</p> <p>S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.</p>
<ul style="list-style-type: none"> <li>Use probability to evaluate outcomes of decisions.</li> </ul> <p><i>This unit sets the stage for work in Algebra II, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.</i></p>	<p>S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).</p> <p>S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).</p>

# Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

## 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

#### **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

#### **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.